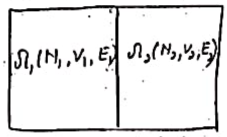
* Baltzmann's Definition of Entropy →

$$S = K_B \ln \Omega$$

Consider two independent system having macrostates are described by (N_1, V_1, E_1) & (N_2, V_2, E_2) with thermodyn. probabilities $\Omega_1(N_1, V_1, E_1)$ & $\Omega_2(N_2, V_2, E_2)$ respectively.



Thermodynamical probability of composition

$$\Omega = \Omega_1 \times \Omega_2 \quad \text{—①}, \quad \text{En. of composition } E = E_1 + E_2 \quad \text{—②}$$

If energy is allowed to exchange

$$\Omega = \Omega_1(E_1) \times \Omega(E_2)$$

Let equilibrium energy is $\bar{E}$

$$\Omega = \max$$

$$\left.\frac{\partial \Omega}{\partial E_1}\right|_{E=\bar{E}} = 0$$

$$0 = \Omega_2 \frac{\partial \Omega_1}{\partial E_1} \frac{\partial E_1}{\partial E_1} + \frac{\partial \Omega_2}{\partial E_2} \frac{\partial E_2}{\partial E_1} \Omega_1$$

$+1$, -1

$$\frac{1}{\Omega_1} \frac{\partial \Omega}{\partial E_1} = \frac{1}{\Omega_2} \frac{\partial \Omega_2}{\partial E_2}$$

$$\frac{1}{\Omega} \frac{\partial \Omega}{\partial E} = \text{constt} \quad \text{—③}$$

$$\frac{\partial}{\partial E}(\ln \Omega) = \text{constt} \quad \text{—④}$$

$$\beta = \frac{1}{K_B T} \qquad \text{S. entropy} = \frac{1}{T}$$

By thermodynamic

$$\left.\frac{\partial S}{\partial E}\right|_{N,V} = \frac{1}{T} \quad \text{—④}$$

[from $TdS = dE + PdV - \mu dN$

Dividing ⑤ by ④

$$\frac{\partial S}{\partial(\ln \Omega)} = \frac{1}{\beta T} = K_B$$

$$S = K_B \ln \Omega + C$$

$$C = 0 \qquad \text{at } T \to 0K, \; S \to 0$$

$$\boxed{S = K_B \ln \Omega}$$

$$\frac{\partial S}{\partial V} = \frac{P}{T}$$

→ If we allow volume to exchange

$$\frac{\partial}{\partial V}(\ln \Omega) = \eta = \frac{P}{K_B T} = \frac{P}{\tau}$$

If allow N to exchange

$$\frac{\partial}{\partial N}(\ln \Omega) = \gamma = -\frac{\mu}{K_B T}$$

$$\left.\frac{\partial S}{\partial E}\right|_{N,V} = \frac{1}{T} \qquad \left.\frac{\partial S}{\partial V}\right|_{N,E} = \frac{P}{T}$$

$\sigma \to$ statistical entropy

$\tau \to$ statistical temp.

$\beta =$ inverse temp.

Th. E. $\boxed{S = K_B \sigma} \to$ S. E.

$$\left.\frac{\partial \sigma}{\partial E}\right|_{N,V} = \frac{1}{\tau} \qquad \left.\frac{\partial \sigma}{\partial V}\right|_{E,N} = \frac{P}{\tau} \qquad \left.\frac{\partial \sigma}{\partial N}\right|_{E,V} = -\frac{\mu}{\tau}$$

Prob: Consider a system of N magnetic atom with spin 1/2. At low temp. the system is ferromagnetic while at high temp. T it is paramagnetic. Neglecting all DOF except spins, find entropy of the system at.

(i) $T \to 0$ (b) $T \to \infty$

(i) $T \to 0K$

Distribution of spins

→ Almost all spins are aligned in same dir^n.

- No. of ways of distribution

$$\Omega \to 1$$

$$S = K_B \ln \Omega$$

$$S \to 0$$

imp

(ii) $T \to \infty$

System attain equilibrium.

$$\Omega = \max.$$

No. of distr. / Max. No. of microstate $= (2S+1)^N = \underbrace{(2 \times \tfrac{1}{2} + 1)}^N = 2^N$

Entropy $S = K_B \ln(2^N)$

$$= (\ln 2^{N K_B})$$

Q. Consider a system of N paramagnetic atoms each have mag. moment M, are placed in mag. field B. n atoms are allined $\parallel^e$ to B & (N−n) allined anti$\parallel^e$ to B. find

(i) Internal En. of the system

(ii) Entropy of the system

(iii) The thermodynamic temp. of the system.

(i) $E = nE_{\parallel^e} + (N-n)E_{anti\parallel^e}$

$$= n(-MB) + (N-n)MB$$

$$E = (N-2n)MB$$

(ii) $\Omega = {}^N C_n = \dfrac{\lfloor N}{\lfloor n \, \lfloor N-n}$

$$S = K_B \ln \Omega$$

$$= K_B \ln \left(\frac{\lfloor N}{\lfloor n \, \lfloor N-n} \right)$$

$$S = N K_B \ln N - N K_B - N K_B \ln n + n K_B - (N-n) K_B \ln(N-n) + (N-n) K_B$$

$$S = K_B \left[N \ln \left(\frac{N}{N-n} \right) - n \ln \left(\frac{n}{N-n} \right) \right]$$

(iii) $\dfrac{1}{T} = \dfrac{\partial S}{\partial E} = \dfrac{\partial S}{\partial n} \cdot \dfrac{\partial n}{\partial E}$

$$= \frac{\partial S}{\partial n} \left(\frac{-1}{2MB} \right)$$

$$= \frac{K_B}{2MB} \ln \left(\frac{n}{N-n} \right)$$

$$T = \frac{2MB}{K_B \ln \left(\frac{n}{N-n} \right)}$$